4.1 - Rational Functions and Asymptotes

Vertical Asymptotes:

1. Find: set den. equal to zero.

\[ f(x) = \frac{1}{x} \]

2. The never crosses V.A.

\[ f(x) = \frac{1}{x - 3} \]

V.A. = \( x = 3 \)

H.A. = \( y = 0 \)
Horizontal Asymptotes ("end behavior")

Compare degrees:

If:  - Degree of numerator is **less than** degree of denominator  
     \( y = 0 \) is a horizontal asymptote

  - Degree of numerator is **greater than** degree of denominator:  
    there are **no** horizontal asymptotes.  
    there is a **slant** asymptote

  - Degree of numerator equals degree of denominator:  
    \( y = \left(\text{leading coefficient of num.}\right) \div \left(\text{leading coefficient of denominator}\right) \)  
    (RATIO OF THE COEFFICIENTS)

Example 4:

\[
\frac{1}{x + 3} = \frac{x + 5}{x + 3}
\]

\[\text{V.A.: } x = -3\]
\[\text{H.A.: } y = 1\]
Example 5:

\[ f(x) = \frac{x}{x^2 - 4} \]

Vertical Asymptote: \( x = \pm 2 \)

Horizontal Asymptote: \( y = 0 \)

Intercepts:
- \( x \)-intercepts: \( x = \pm 2 \)
- \( y \)-intercept: \( y = 0 \)
Roots - What are they?

How do we find them with Rational Functions?

What does this tell us about the graph?

Example 7:

\[ f(x) = \frac{x^2 - 5x + 6}{x^2 - 5x - 6} \]

\[ f(x) = \frac{(x-3)(x-2)}{(x-6)(x+1)} \]

V. A.: \( x = 6, -1 \)

H. A.: \( y = 1 \)

Roots: \( x = 3, 2 \)
\[ f(x) = \frac{x^2 - 5x + 6}{x^2 - 5x - 6} \]

\[ f(x) = (x-3)(x-2) \]

\[ \text{Vertical Asymptote: } x = 6, -1 \]

\[ \text{Horizontal Asymptote: } y = 1 \]

\[ \text{Roots: } x = 3, 2 \]

\[ \text{X-intercepts: } x = 6, -1 \]